Turbo Code Performance as a Function of Code Block Size

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Abstract — **We show that a single family of turbo codes is "nearly perfect" with respect to Shannon's sphere packing bound, over a wide range of code rates and block sizes. We also assess the "imperfectness" of various non-turbo codes for comparison.**

Excitement about turbo codes [1] was sparked by their close approach to ultimate performance limits dictated by channel capacity. For block sizes on the order of 10^4 bits and higher, the required bit-signal-to-noise ratio (SNR) E_b/N_0 for a turbo code of rate r closely approaches the capacity limit for codes constrained to rate r. For smaller block sizes, the required E_b/N_0 strays farther from the rate-constrained capacity limit. However, just as a constraint on r raises the capacity-limited E_b/N_0, so does a constraint on code block size n or information block size k. A classic lower bound on the error probability for codes of specific k and r is Shannon's sphere-packing bound [2]. We evaluated this bound for equal-energy signals applied to a continuous-input additive white Gaussian noise (AWGN) channel; Fig. 1 shows the minimum E_b/N_0 to attain $P_w = 10^{-4}$.

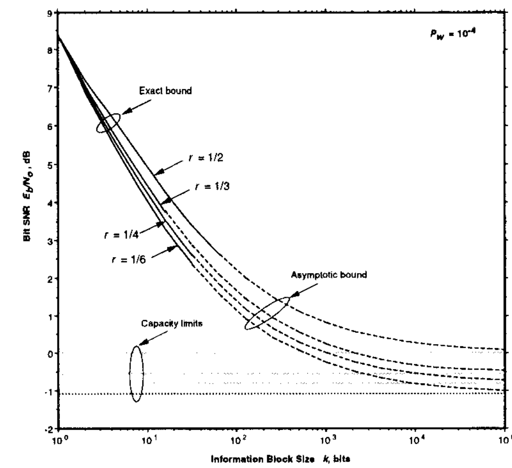

Fig. 1: Sphere packing bound for the continuous-input AWGN channel.

The sphere-packing bound would be reached with equality only if the code were a *perfect* code for this channel, i.e., if equal-size non-intersecting cones could be drawn around every codeword to completely fill n-dimensional space. We define the *imperfectness* of a given code as the difference between the code's required E_b/N_0 to attain a given P_w, and the minimum possible E_b/N_0 required to attain the same P_w, implied by the sphere-packing bound for codes with the same block size k and code rate r. These differences, measured in dB, are shown in Fig. 2 for various codes, with $P_w = 10^{-4}$.

The simulated turbo codes in this figure are systematic parallel concatenated codes constructed from two recursive convolutional components with constraint length 5. The imperfectness of these turbo codes is approximately 0.7 dB for all four code rates plotted, for all block sizes above 1000 bits.

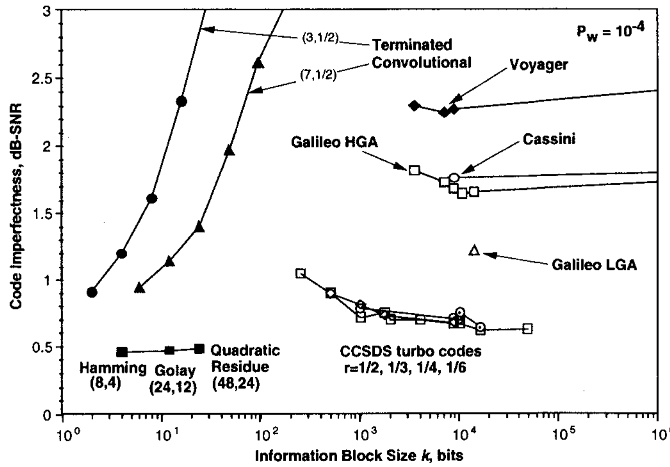

Fig. 2: Imperfectness of various codes relative to the sphere packing bound.

Fig. 2 also shows the imperfectness of various "families" of non-turbo codes for comparison: (a) a family of "best-d_{min}" codes of rate 1/2 (with maximum likelihood decoding); (b) two families of rate-1/2 convolutional codes terminated to various block lengths; and (c) four families of concatenated codes used in JPL's deep space missions over the past two decades. The concatenated codes used inner convolutional codes of various rates and constraint lengths, and an outer code block consisting of interleaved (255,223) Reed-Solomon codewords. Each concatenated code family in the figure is obtained by keeping the component codes constant and varying the interleaving depth. One concatenated code (Galileo LGA) used a variable-redundancy (255,x) outer code and 4-stage iterative decoding; results are shown only for a fixed interleaving depth.

Fig. 2 shows that JPL's long codes historically marched toward perfectness in roughly half-dB steps from Voyager to Cassini to Galileo LGA to future missions that will use turbo codes. The turbo codes' 0.7 dB of imperfectness is not unprecedented. The three classic "best-d_{min}" codes, designed to maximize minimum Hamming distance, approach perfectness even more closely (within 0.5 dB). However, never before turbo codes have there been practically decodable, nearly perfect codes with block sizes beyond a few tens of bits. For example, the terminated convolutional codes in Fig. 2 are nearly perfect (just under 1 dB of imperfectness) only at their smallest possible block sizes. In contrast, the turbo code family is uniformly nearly perfect for all block sizes above 1000 bits. More detailed results are in [3].

References

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